FID201 Modern Physics

Problem Set 1

Probability, Wave Function, and Uncertainty Principle

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1. Suppose you drop a rock off a cliff of height h. As it falls, you snap a million photographs, at random intervals. On each picture you measure the distance the rock has fallen. Question: What is the average of the distance traveled?

   (Hint: the rock is falling freely (no initial speed) so the position function is $y(t) = \frac{1}{2}\, g\, t^2$. First find the PDF by definition, check the normalization, and finally calculate the expectation value of x.)

2. Consider the first 25 digits in the decimal expansion of π $(3, 1, 4, 1, 5, 9, \ldots)$.

   (a) If you selected one number at random from this set, what are the probabilities of getting each of the 10 digits.

   (b) What is the most probable digit? What is the median digit? What is the average value?

   (c) Find the standard deviation for this distribution!

3. Consider the Gaussian distribution

$$\rho(x) = A\ \exp\left(-\lambda\,(x-a)^2\right),$$

   where A, a, and λ are constants.

   (a) If Gaussian distribution is normalized, determine A.

   (b) Find the expectation value of x and x^2, and standard deviation σ.

   (c) Sketch the graph of $\rho(x)$.

4. Following the Born's statistical interpretation and superposition principle, an electron can be presented as the following wave function:

$$\Psi = a\,\Psi_1 + b\,\Psi_2\ .$$

What is the probability that electron in state Ψ_1 and Ψ_2?

5. Calculate

$$\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}\ .$$

This is known as Ehrenfest's theorem – it tells us that the *expectation values* obey Newton's second law.

6. A particle of mass m is in the state

$$\Psi(t,x) = A\ \exp\left(-a\left(\frac{m\,x^2}{\hbar} + \Im t\right)\right),$$

where A and a are positive real constants.

(a) Find A.

(b) Calculate the expectation values of x, x^2, p, and p^2.

(c) Find σ_x and σ_p. Is their product consisten with the uncertainty principle?

7. At time $t = 0$ a particle is represented by the wave function

$$\Psi(0,x) = \begin{cases} A\,(a^2 - x^2) & \text{if } -a \le x \le +a\ , \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the normalization constant A.

(b) Sketch $\Psi(0,x)$ as a function of x.

(c) What is the expectation value of x (at time $t = 0$).

(d) Find is the expectation value of p (at time $t = 0$).Note: You *cannot* get it from $m\,\mathrm{d}\langle x\rangle/\mathrm{d}t$, why?

(e) Check whether the particle still obeys the Heisenberg's uncertainty principle.